General analysis of CP violation in polarized $b \to d\ell^+\ell^-$ decay

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Abstract

The CP violating asymmetries in the $b \to d\ell^+\ell^-$ decay, when one of the leptons is polarized, is investigated using the most general form of the effective Hamiltonian. The sensitivity of the CP violating asymmetries on the new Wilson coefficients is studied.

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1   Introduction

Rare B meson decays, induced by the flavor changing neutral current (FCNC) $b \to s(d)$ transitions, provide one the most promising research area in particle physics. Interest to rare B meson decays has its roots in their role being potentially the precision testing ground for the standard model (SM) at loop level and looking for new physics beyond the SM [1]. Experimentally these decays will provide a more precise determination of the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, such as, V_{tq} $(q = d, s, b)$, V_{ub} and CP violation. In FCNC decays, any deviation over the SM results is an unambiguous indication for new physics. The first observation of the radiative $B \to X_s \gamma$ decay by CLEO [2], and later by ALEPH [3], have yielded $|V_{tb} V_{ts}^*| \sim 0.035$, which is in confirmation with the CKM estimates.

Rare semileptonic decays $b \to s(d) \ell^+ \ell^-$ can provide alternative sources for searching new physics beyond the SM, and these decays are relatively clean compared to pure hadronic decays. The matrix elements of the $b \to s \ell^+ \ell^-$ transition contain terms describing the virtual effects induced by the $t\bar{t}$, $c\bar{c}$ and $u\bar{u}$ loops, which are proportional to $|V_{tb} V_{ts}^*|$, $|V_{cb} V_{cs}^*|$ and $|V_{ub} V_{us}^*|$, respectively.

Using the unitarity condition of the CKM matrix and neglecting $|V_{ub} V_{us}^*|$ in comparison to $|V_{tb} V_{ts}^*|$ and $|V_{cb} V_{cs}^*|$, it is obvious that, the matrix element for the $b \to s \ell^+ \ell^-$ decay involves only one independent CKM matrix element, namely, $|V_{tb} V_{ts}^*|$, so that CP–violation in this channel is strongly suppressed in the SM.

As has already been noted, $b \to q \ell^+ \ell^-$ decay is a promising candidate for establishing new physics beyond the SM. New physics effects manifest themselves in rare B decays in two different ways; either through new contributions to the Wilson coefficients existing in the SM, or through the new structures in the effective Hamiltonian, which are absent in the SM. Note that, the semileptonic $b \to q \ell^+ \ell^-$ decay has extensively been studied in numerous works [4]–[19], in the framework of the SM and its various extensions. Recently, the first measurement of the $b \to s \ell^+ \ell^-$ decay has been reported by BELLE [20] and is in agreement with the SM expectation. Therefore, this result puts further constraint on any extension of the SM.

In the present work we examine CP violating effects for the case when one of the leptons is polarized, in model independent framework, by taking into account a more general form of the effective Hamiltonian. It should be noted here that similar calculation has been carried out in the SM in [21].

One efficient way in establishing new physics beyond the SM is the measurement of the lepton polarization [22]–[33]. This issue has been studied for the $b \to s \tau^+ \tau^-$ transition and $B \to K^* \ell^+ \ell^-$, $B \to K \ell^+ \ell^-$ decays in a model independent way in [27] and [32, 33], respectively.

The paper is organized as follows. In section 2, using the most general form of the four–Fermi interaction, we derive model independent expressions for the CP violating asymmetry, for polarized and unpolarized leptons. In section 3 we present our numerical analysis.

2   The formalism

In this section we present the necessary expressions for CP violating asymmetry when lepton is polarized and unpolarized, using the most general form of four–Fermi interactions. Following [25, 27], we write the matrix element of the $b \rightarrow d\ell^+\ell^-$ transition in terms of the twelve model independent four–Fermi interactions

$$
\begin{aligned}
\mathcal{M} = \frac{G\alpha}{\sqrt{2}\pi} V_{tb}V_{td}^* \Big\{ & C_{SL}\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Lb\bar{\ell}\gamma^\mu\ell + C_{BR}\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Rb\bar{\ell}\gamma^\mu\ell + C_{LL}^{tot}\bar{d}_L\gamma_\mu b_L\bar{\ell}_L\gamma^\mu\ell_L \\
& + C_{LR}^{tot}\bar{d}_L\gamma_\mu b_L\bar{\ell}_R\gamma^\mu\ell_R + C_{RL}\bar{d}_R\gamma_\mu b_R\bar{\ell}_L\gamma^\mu\ell_L + C_{RR}\bar{d}_R\gamma_\mu b_R\bar{\ell}_R\gamma^\mu\ell_R \\
& + C_{LRLR}\bar{d}_Lb_R\bar{\ell}_L\ell_R + C_{RLLR}\bar{d}_Rb_L\bar{\ell}_L\ell_R + C_{LRRL}\bar{d}_Lb_R\bar{\ell}_R\ell_L + C_{RLRL}\bar{d}_Rb_L\bar{\ell}_R\ell_L \\
& + C_T\bar{d}\sigma_{\mu\nu}b\bar{\ell}\sigma^{\mu\nu}\ell + iC_{TE}\epsilon_{\mu\nu\alpha\beta}\bar{d}\sigma^{\mu\nu}b\bar{\ell}\sigma^{\alpha\beta}\ell \Big\} \, ,
\end{aligned} \tag{1}
$$

where L and R stand for the chiral operators $L = (1-\gamma_5)/2$ and $R = (1+\gamma_5)/2$, respectively, and C_X are the coefficients of the four–Fermi interactions. The first two terms, C_{SL} and C_{BR} are the nonlocal Fermi interactions which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The next four terms are the vector type interactions with coefficients C_{LL}^{tot}, C_{LR}^{tot}, C_{RL} and C_{RR}. Two of these vector interactions containing the coefficients C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in the forms $C_9^{eff} - C_{10}$ and $C_9^{eff} + C_{10}$, respectively. Therefore C_{LL}^{tot} and C_{LR}^{tot} can be represented as

$$
\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} \, , \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} \, ,
\end{aligned}
$$

where C_{LL} and C_{LR} describe the contributions of new physics. The following four terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions, and the last two terms with the coefficients C_T and C_{TE} are the tensor type interactions. It should be noted here that, in further analysis we will assume that all new Wilson coefficients are real, as is the case in the SM, while only C_9^{eff} contains imaginary part and it is parametrized in the following form

$$
C_9^{eff} = \xi_1 + \lambda_u\xi_2 \, , \tag{2}
$$

where

$$
\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \, ,
$$

and

$$
\begin{aligned}
\xi_1 &= 4.128 + 0.138\omega(\hat{s}) + g(\hat{m}_c, \hat{s})C_0(\hat{m}_b) - \frac{1}{2}g(\hat{m}_d, \hat{s})(C_3 + C_4) \\
& - \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) \, , \\
\xi_2 &= [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})](3C_1 + C_2) \, ,
\end{aligned} \tag{3}
$$

where $\hat{m}_q = m_q/m_b$, $\hat{s} = q^2$, $C_0(\mu) = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$, and

$$\omega(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(\hat{s}) - \frac{2}{3}\ln(\hat{s})\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s})$$
$$- \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln(\hat{s}) + \frac{5+9\hat{s}-6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})} , \tag{4}$$

represents the $O(\alpha_s)$ correction coming from one gluon exchange in the matrix element of the operator $\mathcal{O}_9$ [34], while the function $g(\hat{m}_q, \hat{s})$ represents one–loop corrections to the four–quark operators O_1–O_6 [35], whose form is

$$g(\hat{m}_q, \hat{s}) = -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2+y_q)$$
$$- \sqrt{|1-y_q|}\left\{\theta(1-y_q)\left[\ln\left(\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}}\right) - i\pi\right] + \theta(y_q-1)\arctan\left(\frac{1}{\sqrt{y_q-1}}\right)\right\} , \tag{5}$$

where $y_q = 4\hat{m}_q^2/\hat{s}$.

In addition to the short distance contributions, $B \to X_d\ell^+\ell^-$ decay also receives long distance contributions, which have their origin in the real $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ intermediate states, i.e., ρ, ω and J/ψ family. There are four different approaches in taking long distance contributions into consideration: a) HQET based approach [36], b) AMM approach [37], c) LSW approach [38], and d) KS approach [33]. In the present work we choose the AMM approach, in which these resonance contributions are parametrized using the Breit–Wigner form for the resonant states. The effective coefficient C_9^{eff} including the ρ, ω and J/ψ resonances are defined as

$$C_9^{eff} = C_9(\mu) + Y_{res}(\hat{s}) , \tag{6}$$

where

$$Y_{res} = -\frac{3\pi}{\alpha^2}\left\{ \left(C^{(0)}(\mu) + \lambda_u\left[3C_1(\mu) + C_2(\mu)\right]\right)\sum_{V_i=\psi} K_i \frac{\Gamma(V_i \to \ell^+\ell^-)M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \right.$$
$$\left. - \lambda_u g(\hat{m}_u, \hat{s})\left[3C_1(\mu) + C_2(\mu)\right]\sum_{V_i=\rho,\omega} \frac{\Gamma(V_i \to \ell^+\ell^-)M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \right\} . \tag{7}$$

The phenomenological factor K_i has the universal value for the inclusive $B \to X_s\ell^+\ell^-$ decay $K_i \simeq 2.3$ [39], which we use in our calculations.

As we have already noted, CP asymmetry can appear both for cases when lepton is polarized or unpolarized, and hence, along this line, we will present the expressions for the differential decay rate for both cases when the lepton is polarized and unpolarized. Starting with Eq. (1), after lengthy calculations we get the following expression for the unpolarized decay width:

$$\frac{d\Gamma}{d\hat{s}} = \frac{G^2\alpha^2 m_b^5}{32768\pi^5}|V_{tb}V_{td}^*|^2 \lambda^{1/2}(1,\hat{s},0)v\Delta(\hat{s}) , \tag{8}$$

where $\hat{s} = q^2/m_b^2$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the velocity of the final lepton, $\hat{m}_\ell = m_\ell/m_b$, and $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$ is the usual triangle function. The explicit expression of the function $\Delta(\hat{s})$ can be written as

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$$\Delta(\hat{s}) = 16(1 - \hat{s}) \operatorname{Re}\left\{ \frac{8}{3\hat{s}^2}(2\hat{m}_\ell^2 + \hat{s})(2 + \hat{s})\left[|C_{BR}|^2 + 8\hat{s}\left(|C_T|^2 + 4|C_{TE}|^2\right)\right] \right.$$

$$- \frac{8}{\hat{s}}(2\hat{m}_\ell^2 + \hat{s})\left(C_{LL}^{tot} + C_{LR}^{tot}\right)C_{BR}^* - \frac{32}{\hat{s}}\hat{m}_\ell(2 + \hat{s})\left(C_T - 2C_{TE}\right)C_{BR}^*$$

$$- \frac{4}{3\hat{s}}[2\hat{s}(\hat{m}_\ell^2 - \hat{s}) - (2\hat{m}_\ell^2 + \hat{s})]\left(|C_{LL}^{tot}|^2 + |C_{LR}^{tot}|^2 + |C_{RL}|^2 + |C_{RR}|^2\right)$$

$$- 2(2\hat{m}_\ell^2 - \hat{s})\left(|C_{LRLR}|^2 + |C_{RLLR}|^2 + |C_{LRRL}|^2 + |C_{RLRL}|^2\right)$$

$$+ 8\hat{m}_\ell^2\left[2\left(C_{LL}^{tot}(C_{LR}^{tot})^* + C_{RL}C_{RR}^*\right) - (C_{LRLR}C_{LRRL}^* + C_{RLLR}C_{RLRL}^*)\right]$$

$$- 4\hat{m}_\ell\left[\left(C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_{LRLR}^* - C_{LRRL}^*\right) + (C_{RL} - C_{RR})\left(C_{RLLR}^* - C_{RLRL}^*\right)\right.$$

$$\left.\left. - 12\left(C_{LL}^{tot} + C_{LR}^{tot}\right)\left(C_T^* - 2C_{TE}^*\right) - 12\left(C_{RL} + C_{RR}\right)\left(C_T^* + 2C_{TE}^*\right)\right]\right\} , \qquad (9)$$

Our result agrees with the one given in [25], except the term multiplying the coefficient $N_9(s)$ in [25]. The differential decay width for the CP conjugated process can be obtained from Eq. (7) by making the replacement

$$\Delta \to \bar{\Delta}, \text{ i.e., } C_9^{eff} = \xi_1 + \lambda_u\xi_2 \to \bar{C}_9^{eff} = \xi_1 + \lambda_u^*\xi_2 .$$

The lepton polarization has been firstly analyzed in the SM in [33] and [40], where it has been shown that additional information can be obtained about the quadratic functions of the Wilson coefficients C_7^{eff}, C_9^{eff} and C_{10}. In order to calculate the final lepton polarization, we define the orthogonal unit vectors $\vec{e}_L$, $\vec{e}_T$ and $\vec{e}_N$ in such a way that, in the rest frame of leptons they can be expressed as

$$s_L^{-\mu} = \left(0, \vec{e}_L^-\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) ,$$

$$s_N^{-\mu} = \left(0, \vec{e}_N^-\right) = \left(0, \frac{\vec{p}_s \times \vec{p}_-}{|\vec{p}_s \times \vec{p}_-|}\right) ,$$

$$s_T^{-\mu} = \left(0, \vec{e}_T^-\right) = \left(0, \vec{e}_N^- \times \vec{e}_L^-\right) ,$$

$$s_L^{+\mu} = \left(0, \vec{e}_L^+\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ,$$

$$s_N^{+\mu} = \left(0, \vec{e}_N^+\right) = \left(0, \frac{\vec{p}_s \times \vec{p}_+}{|\vec{p}_s \times \vec{p}_+|}\right) ,$$

$$s_T^{+\mu} = \left(0, \vec{e}_T^+\right) = \left(0, \vec{e}_N^+ \times \vec{e}_L^+\right) ,$$

where $\vec{p}_-$, $\vec{p}_+$ and $\vec{p}_s$ are the three–momenta of the leptons ℓ^-, ℓ^+, and the strange quark in the center of mass frame (CM) of $\ell^-\ell^+$, respectively, and the subscripts L, N and T stand for the longitudinal, normal and transversal polarization of the lepton. Boosting the unit vectors s_L^- and s_L^+ corresponding to longitudinal polarization by Lorentz transformation, from the rest frame of the corresponding leptons, to the $\ell^-\ell^+$ CM frame, we get

$$\left(s_L^{-\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell|\vec{p}_-|}\right) ,$$

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$$\left(s_L^{+\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell} , -\frac{E\vec{p}_-}{m_\ell |\vec{p}_-|}\right) , \tag{10}$$

while $s_N^{\mp\mu}$ and $s_T^{\mp\mu}$ are not changed by the boost.

The differential decay rate of the $b \to d\ell^+\ell^-$ decay, for any spin direction $\vec{n}^{\mp}$ of $\ell^{\mp}$, where $\vec{n}^{\mp}$ is the unit vector in the rest frame of $\ell^{\mp}$, can be written in the following form

$$\frac{d\Gamma(s, \vec{n}^{\mp})}{d\hat{s}} = \frac{1}{2}\frac{d\Gamma}{d\hat{s}}_0 \left[1 + \left(P_L^{\mp}\vec{e}_L^{\mp} + P_N^{\mp}\vec{e}_N^{\mp} + P_T^{\mp}\vec{e}_T^{\mp}\right) \cdot \vec{n}^{\mp}\right] , \tag{11}$$

where $(d\Gamma/d\hat{s})_0$ corresponds to the unpolarized differential decay width (see Eq. (8)) for the $b \to d\ell^+\ell^-$ decay. The differential decay width for the $\bar{b} \to \bar{d}\ell^+\ell^-$ decay, can simply be obtained by making the replacement

$$\frac{d\Gamma(s, \vec{n}^{\mp})}{d\hat{s}} \to \frac{d\bar{\Gamma}(s, \vec{n}^{\mp})}{d\hat{s}} ,$$

where $d\bar{\Gamma}/d\hat{s}$ is obtained from $d\Gamma/d\hat{s}$ by replacing $C_9^{eff} = \xi_1 + \lambda_u\xi_2$ to $\bar{C}_9^{eff} = \xi_1 + \lambda_u^*\xi_2$. The polarizations P_L, P_N and P_T are defined as

$$P_i^{\mp}(\hat{s}) = \frac{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp} = \vec{e}_i^{\mp}) - \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp} = -\vec{e}_i^{\mp})}{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp} = \vec{e}_i^{\mp}) + \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp} = -\vec{e}_i^{\mp})} = \frac{\Delta_i^{\mp}}{\Delta} , \tag{12}$$

with $i = L,\ N,\ T$.

The explicit expressions for the longitudinal polarization asymmetries P_L^- and P_L^+ are

$$\begin{aligned}
P_L^- = \frac{32(1-\hat{s})v}{\Delta}\, \text{Re}\Big\{&4\left(C_{LL}^{tot} - C_{LR}^{tot}\right)C_{BR}^* \\
&-\frac{2}{3}(1+2\hat{s})\left(|C_{LL}^{tot}|^2 - |C_{LR}^{tot}|^2 + |C_{RL}|^2 - |C_{RR}|^2 - 128C_TC_{TE}^*\right) \\
&+\frac{16}{3\hat{s}}\hat{m}_\ell(2+\hat{s})\left(C_T - 2C_{TE}\right)C_{BR}^* + 2\hat{m}_\ell\Big[\left(C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_{LRLR}^* + C_{LRRL}^*\right) \\
&+ (C_{RL} - C_{RR})\left(C_{RLLR}^* + C_{RLRL}^*\right) - 4\left(3C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_T^* - 2C_{TE}^*\right) \\
&- 4\left(C_{RL} - 3C_{RR}\right)\left(C_T^* + 2C_{TE}^*\right)\Big] - \hat{s}\Big(|C_{LRLR}|^2 - |C_{LRRL}|^2 \\
&+ |C_{RLLR}|^2 - |C_{RLRL}|^2 + 128C_T^*C_{TE}\Big)\Big\},
\end{aligned}$$

and

$$\begin{aligned}
P_L^+ = \frac{32(1-\hat{s})v}{\Delta}\, \text{Re}\Big\{&-4\left(\bar{C}_{LL}^{tot} - \bar{C}_{LR}^{tot}\right)C_{BR}^* \\
&+\frac{2}{3}(1+2\hat{s})\left(|\bar{C}_{LL}^{tot}|^2 - |\bar{C}_{LR}^{tot}|^2 + |C_{RL}|^2 - |C_{RR}|^2 + 128C_TC_{TE}^*\right)
\end{aligned}$$

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$$
+ \frac{16}{3\hat{s}} \hat{m}_\ell (2 + \hat{s}) (C_T - 2C_{TE}) C_{BR}^* + 2\hat{m}_\ell \Big[\left(\bar{C}_{LL}^{tot} - \bar{C}_{LR}^{tot} \right) (C_{LRLR}^* + C_{LRRL}^*)
$$

$$
+ (C_{RL} - C_{RR})(C_{RLLR}^* + C_{RLRL}^*) + 4 \left(\bar{C}_{LL}^{tot} - 3\bar{C}_{LR}^{tot} \right) (C_T^* - 2C_{TE}^*)
$$

$$
+ 4 (3C_{RL} - C_{RR})(C_T^* + 2C_{TE}^*) \Big] - \hat{s} \Big(|C_{LRLR}|^2 - |C_{LRRL}|^2
$$

$$
+ |C_{RLLR}|^2 - |C_{RLRL}|^2 + 128 C_T^* C_{TE} \Big) \Big\}.
$$

The normal asymmetries, P_N^- and P_N^+, are;

$$
P_N^- = -\frac{4\pi(1 - \hat{s})v\sqrt{\hat{s}}}{\Delta} \, \text{Im}\Big\{ -\frac{8}{\hat{s}} \hat{m}_\ell \left(C_{LL}^{tot} - C_{LR}^{tot} \right) C_{BR}^*
$$

$$
+ 8\hat{m}_\ell \Big[C_{LL}^{tot}(C_{LR}^{tot})^* - C_{RL}C_{RR}^* - 2(C_{LRLR} + C_{LRRL})(C_T^* - 2C_{TE}^*)
$$

$$
- 2(C_{RLLR} + C_{RLRL})(C_T^* + 2C_{TE}^*) \Big] + 4\Big[(C_{LRLR} + C_{LRRL}) C_{BR}^*
$$

$$
+ C_{LL}^{tot}C_{LRRL}^* + C_{LR}^{tot}C_{LRLR}^* + C_{RL}C_{RLRL}^* + C_{RR}C_{RLLR}^* - 4C_{RL}(C_T^* + 2C_{TE}^*)
$$

$$
- 4C_{LR}^{tot}(C_T^* - 2C_{TE}^*) \Big] - \frac{16}{\hat{s}}(C_T - 2C_{TE}) C_{BR}^* \Big\} \,,
$$

$$
P_N^+ = -\frac{4\pi(1 - \hat{s})v\sqrt{\hat{s}}}{\Delta} \, \text{Im}\Big\{ \frac{8}{\hat{s}} \hat{m}_\ell \left(\bar{C}_{LL}^{tot} - \bar{C}_{LR}^{tot} \right) C_{BR}^*
$$

$$
+ 8\hat{m}_\ell \Big[- \bar{C}_{LL}^{tot}(\bar{C}_{LR}^{tot})^* + C_{RL}C_{RR}^* + 2(C_{LRLR} + C_{LRRL})(C_T^* - 2C_{TE}^*)
$$

$$
+ 2(C_{RLLR} + C_{RLRL})(C_T^* + 2C_{TE}^*) \Big] - 4\Big[(C_{LRLR} + C_{LRRL}) C_{BR}^*
$$

$$
+ \bar{C}_{LL}^{tot}C_{LRLR}^* + \bar{C}_{LR}^{tot}C_{LRRL}^* + C_{RL}C_{RLLR}^* + C_{RR}C_{RLRL}^* + 4\bar{C}_{LL}^{tot}(C_T^* - 2C_{TE}^*)
$$

$$
+ 4C_{RR}(C_T^* + 2C_{TE}^*) \Big] + \frac{16}{\hat{s}}C_{BR}(C_T^* - 2C_{TE}^*) \Big\} \,.
$$

The transverse asymmetries, P_T^- and P_T^+, are;

$$
P_T^- = -\frac{8\pi(1 - \hat{s})}{\Delta\sqrt{\hat{s}}} \, \text{Re}\Big\{ -\frac{8}{\hat{s}} \hat{m}_\ell |C_{BR}|^2 + 4\hat{m}_\ell \left(3C_{LL}^{tot} + C_{LR}^{tot} \right) C_{BR}^*
$$

$$
- 2(1 + \hat{s})\hat{m}_\ell \left(|C_{LL}^{tot}|^2 - |C_{RR}|^2 \right) + 4\hat{m}_\ell\hat{s}\Big[- C_{LL}^{tot}(C_{LR}^{tot})^* + C_{RL}C_{RR}^*
$$

$$
+ 2(C_{LRLR} - C_{LRRL})(C_T^* - 2C_{TE}^*) + 2(C_{RLLR} - C_{RLRL})(C_T^* + 2C_{TE}^*) \Big]
$$

$$
+ 2(1 - \hat{s})\hat{m}_\ell \left(|C_{LR}^{tot}|^2 - |C_{RL}|^2 \right) - 2\hat{s}(C_{LRLR} - C_{LRRL}) C_{BR}^*
$$

$$
+ \frac{8}{\hat{s}}(4\hat{m}_\ell^2 + \hat{s})C_{BR}(C_T^* - 2C_{TE}^*) + 4\hat{m}_\ell^2\Big[C_{LL}^{tot}C_{LRLR}^* - C_{LR}^{tot}C_{LRRL}^*
$$

$$
+ C_{RL}C_{RLLR}^* - C_{RR}C_{RLRL}^* - 12C_{LL}^{tot}(C_T^* - 2C_{TE}^*) + 12C_{RR}(C_T^* + 2C_{TE}^*) \Big]
$$

$$
+ 2(2\hat{m}_\ell^2 - \hat{s})\Big[C_{LL}^{tot}C_{LRRL}^* - C_{LR}^{tot}C_{LRLR}^* + C_{RL}C_{RLRL}^* - C_{RR}C_{RLLR}^*
$$

$$
+ 4C_{LR}^{tot}(C_T^* - 2C_{TE}^*) - 4C_{RL}(C_T^* + 2C_{TE}^*) \Big] + 256\hat{m}_\ell C_T C_{TE}^* \Big\} \,,
$$

and

$$P_T^+ = -\frac{8\pi(1-\hat{s})}{\Delta\sqrt{\hat{s}}} \, \mathrm{Re} \Bigg\{ -\frac{8}{\hat{s}}\hat{m}_\ell |C_{BR}|^2 + 4\hat{m}_\ell \left(\bar{C}_{LL}^{tot} + 3\bar{C}_{LR}^{tot} \right) C_{BR}^*$$

$$+ 2(1-\hat{s})\hat{m}_\ell \left(|\bar{C}_{LL}^{tot}|^2 - |C_{RR}|^2 \right) - 4\hat{m}_\ell \hat{s} \Big[\bar{C}_{LL}^{tot}(\bar{C}_{LR}^{tot})^* - C_{RL}C_{RR}^*$$

$$+ 2\left(C_{LRLR} - C_{LRRL} \right)\left(C_T^* - 2C_{TE}^* \right) + 2\left(C_{RLLR} - C_{RLRL} \right)\left(C_T^* + 2C_{TE}^* \right) \Big]$$

$$- 2(1+\hat{s})\hat{m}_\ell \left(|\bar{C}_{LR}^{tot}|^2 - |C_{RL}|^2 \right) + 2\hat{s}\left(C_{LRLR} - C_{LRRL} \right)C_{BR}^*$$

$$+ \frac{8}{\hat{s}}(4\hat{m}_\ell^2 + \hat{s})C_{BR}\left(C_T^* - 2C_{TE}^* \right) + 4\hat{m}_\ell^2 \Big[\bar{C}_{LL}^{tot}C_{LRRL}^* - \bar{C}_{LR}^{tot}C_{LRLR}^*$$

$$+ C_{RL}C_{RLRL}^* - C_{RR}C_{RLLR}^* - 12\bar{C}_{LR}^{tot}\left(C_T^* - 2C_{TE}^* \right) + 12C_{RL}\left(C_T^* + 2C_{TE}^* \right) \Big]$$

$$+ 2(2\hat{m}_\ell^2 - \hat{s})\Big[\bar{C}_{LL}^{tot}C_{LRLR}^* - \bar{C}_{LR}^{tot}C_{LRRL}^* + C_{RL}C_{RLLR}^* - C_{RR}C_{RLRL}^*$$

$$+ 4\bar{C}_{LL}^{tot}\left(C_T^* - 2C_{TE}^* \right) - 4C_{RR}\left(C_T^* + 2C_{TE}^* \right) \Big] + 256\hat{m}_\ell C_T C_{TE}^* \Bigg\} \, .$$

It should be noted here that, these polarizations were calculated in [25], using the most general form of the effective Hamiltonian. Our results for P_L and P_N agree with the ones given in [25], while the transversal polarizations P_T^- and P_T^+ both differ from the ones given in the same work. In the SM case, our results for P_L, P_N and P_T coincide with the results of [21]. It is quite obvious from the expressions of P_i that, they involve various quadratic combinations of the Wilson coefficients and hence they are quite sensitive to the new physics. The polarizations P_N and P_T are proportional to m_ℓ and therefore can be significant for τ lepton only.

Having obtained all necessary expressions, we can proceed now to study the CP violating asymmetries. In the unpolarized lepton case, the CP violating differential decay width asymmetry is defined as

$$A_{CP}(\hat{s}) = \frac{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 - \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} = \frac{\Delta - \bar{\Delta}}{\Delta + \bar{\Delta}} \, , \tag{13}$$

where

$$\frac{d\Gamma}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-)}{d\hat{s}}, \text{ and, } \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\bar{\Gamma}(b \to d\ell^+\ell^-)}{d\hat{s}} \, ,$$

and $(d\bar{\Gamma}/d\hat{s})_0$ can be obtained from $(d\Gamma/d\hat{s})_0$ by making the replacement

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2 \to \bar{C}_9^{eff} = \xi_1 + \lambda_u^* \xi_2 \, . \tag{14}$$

Using Eqs. (11) and (13), we get for the CP violating asymmetry

$$A_{CP}(\hat{s}) = -4\mathrm{Im}[\lambda_u]\frac{\Sigma(\hat{s})}{\Delta(\hat{s}) + \bar{\Delta}(\hat{s})} \, ,$$

$$\approx -2\mathrm{Im}[\lambda_u]\frac{\Sigma(\hat{s})}{\Delta(\hat{s})} \, , \tag{15}$$

and $\Sigma(\hat{s})$, whose explicit form we do not present, can easily be obtained using Eqs. (9) and (13).

In the presence of the lepton polarization CP asymmetry is modified and the source of this modification can be attributed to the presence of new interference terms which contain C_9^{eff} (in our case C_{LL}^{tot} and C_{LR}^{tot}). We now proceed to calculate this new contribution.

In the polarized lepton case, CP asymmetry can be defined as

$$A_{CP}(\vec{s}) = \frac{\dfrac{d\Gamma}{d\hat{s}}(\hat{s}, \vec{n}^-) - \dfrac{d\bar{\Gamma}}{d\hat{s}}(\hat{s}, \vec{n}^+)}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \ , \tag{16}$$

where

$$\frac{d\Gamma}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-(\vec{n}^-))}{d\hat{s}}, \ \text{and,} \ \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+(\vec{n}^+)\ell^-)}{d\hat{s}} \ .$$

The differential decay width with lepton polarization for the $b \to d\ell^+\ell^-$ channel is given by Eq. (11). Analogously, for the corresponding CP conjugated process we have the expression

$$\frac{d\bar{\Gamma}}{d\hat{s}}(\vec{n}^\mp) = \frac{1}{2}\left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0 \left[1 + P_i^+(\vec{e}_i^\mp \cdot \vec{n}^\mp)\right] \ . \tag{17}$$

With the choice $\vec{n}^- = \vec{n}^+$, P_i^+ can be constructed from the differential decay width analogous to Eq. (12). At this stage we have all necessary ingredients for calculation of the CP violating asymmetry for the lepton ℓ^- with polarization $\vec{n} = \vec{e}_i$. Inserting Eqs. (11) and (17) into Eq. (16), and setting $\vec{n}^- = \vec{n}^+$, the CP violating asymmetry when lepton is polarized is defined as

$$A_{CP} = \frac{\dfrac{1}{2}\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 \left[1 + P_i^-(\vec{e}_i^- \cdot \vec{n})\right] - \dfrac{1}{2}\left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0 \left[1 + P_i^+(\vec{e}_i^+ \cdot \vec{n})\right]}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \ .$$

Taking into account the fact that $\vec{e}_{L,N}^+ = -\vec{e}_{L,N}^-$, and $\vec{e}_T^+ = \vec{e}_T^-$, we obtain

$$A_{CP} = \frac{1}{2}\left\{ \frac{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 - \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \pm \frac{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 P_i^- \mp \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0 P_i^+}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \right\} \ . \tag{18}$$

Using Eq. (8), we get from Eq. (18),

$$A_{CP}(\vec{n} = \mp\vec{e}_i^-) = \frac{1}{2}\left\{ \frac{\Delta - \bar{\Delta}}{\Delta + \bar{\Delta}} \pm \frac{\Delta_i \mp \bar{\Delta}_i}{\Delta + \bar{\Delta}} \right\} \ ,$$

$$= \frac{1}{2}\left\{ A_{CP}(\hat{s}) \pm \delta A_{CP}^i(\hat{s}) \right\} \ , \tag{19}$$

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where the upper sign in the definition of δA_{CP} corresponds to L and N polarizations, while the lower sign corresponds to T polarization.

The $\delta A_{CP}^i(\hat{s})$ terms in Eq. (19) describe the modification to the unpolarized decay width, which can be written as

$$
\begin{aligned}
\delta A_{CP}^i(\hat{s}) &= \frac{-4\mathrm{Im}\lambda_u \delta\Sigma^i(\hat{s})}{\Delta(\hat{s}) + \bar{\Delta}(\hat{s})} \ , \\
&\approx -2\mathrm{Im}\lambda_u \frac{\delta\Sigma^i(\hat{s})}{\Delta(\hat{s})} \ .
\end{aligned}
\tag{20}
$$

We do not present the explicit forms of the expressions for $\delta\Sigma^i(\hat{s})$, $(i = L, N, T)$, since their calculations are straightforward.

3 Numerical analysis

In this section we will study the dependence of the CP asymmetries $A_{CP}(\hat{s})$ and $\delta A_{CP}^i(\hat{s})$ on $\hat{s}$ at fixed values of the new Wilson coefficients. Once again, we remind the reader that in the present work all new Wilson coefficients are taken to be real. The experimental result on $b \to s\gamma$ decay put strong restriction on C_{BR}, i.e., practically it has the same value as it has in the SM. Therefore, in further numerical analysis we will set $C_{BR} = -2C_7^{eff}$. Throughout numerical analysis, we will vary all new Wilson coefficients in the range $-4 \leq C_X \leq 4$. The experimental bounds on the branching ratios of the $B \to K(K^*)\mu^+\mu^-$ [41, 42] and $B \to \mu^+\mu^-$ [43] suggest that this is the right order of magnitude for vector and scalar type interactions. Recently, BaBar and BELLE collaborations [40, 41] have presented new results on the branching ratios of $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays. When these results are used, stronger restrictions are imposed on some of the new Wilson coefficients. For example, $-2 \leq C_{LL} \leq 0$, $0 \leq C_{RL} \leq 2.3$, $-1.5 \leq C_T \leq 1.5$ and $-3.3 \leq C_{TE} \leq 2.6$, and rest of the coefficients vary in the region $-4 \leq C_X \leq 4$. However, since the results of BaBar and BELLE are preliminary we will not take into account these results in the present analysis and vary all of the new Wilson coefficients in the region $4 \leq C_X \leq 4$. Furthermore, in our analysis we will use the Wolfenstein parametrization [44] for the CKM matrix. The currently allowed range for the Wolfenstein parameters are: $0.19 \leq \rho \leq 0.268$ and $0.19 \leq \eta \leq 0.268$ [45], where in the present analysis they are set to $\rho = 0.25$ and $\eta = 0.34$.

We start our numerical analysis by first discussing the dependence of A_{CP} on $\hat{s}$ at fixed values of C_i, i.e., $C_i = -4; 0; 4$ which can be summarized as follows

- For the $b \to d\mu_+\mu^-$ case, far from resonance regions, A_{CP} depends strongly on C_{LL}. We observe that, when C_{LL} is positive (negative), the value of A_{CP} decreases (increases), while the situation for the C_{LR} case is the opposite way around (see Figs. (1) and (2)). If the tensor interaction is taken into account, A_{CP} practically seems to be zero for all values of C_T and C_{TE}. Furthermore, A_{CP} shows quite a weak dependence on all remaining Wilson coefficients and the departure from the SM result is very small.

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- For the $b \to d\tau_+\tau^-$ case, in the region between second and third ψ resonances, A_{CP} is sensitive to C_{LR}, C_{LRLR}, C_{LRRL}, C_T, and C_{TE}, as can be seen from the Figs (3), (4), (5), (6) and (7), respectively. When C_{LR} and C_{LRLR} are positive (negative), they contribute destructively (constructively) to the SM result. The situation is contrary to this behavior for the C_{LRRL} scalar coupling. In the tensor interaction case, in the second and third resonance region, the magnitude of A_{CP} is smaller compared to that of the SM result. But, it is quite important to observe that A_{CP} asymmetry changes its sign, compared to its behavior in the SM, when C_T (C_{TE}) is negative (positive). Therefore, determination of the sign and magnitude of A_{CP} can give promising information about new physics.

The results concerning δA_{CP} for the $b \to d\mu^+\mu^-$ decay can be summarized as follows:

- In the region $1 \ GeV^2/m_b^2 \leq \hat{s} \leq 8 \ GeV^2/m_b^2$, which is free of resonance contribution, CP asymmetry due to the longitudinal polarization of μ lepton is dependent strongly on C_{LL}, and is practically independent of all remaining vector interaction coefficients. When C_{LL} is negative (positive), δA_{CP} is larger (smaller) compared to that of the SM result (see Fig. (8)).

- δA_{CP}^L depends strongly on all scalar type interactions. As an example we present the dependence of δA_{CP} on C_{LRRL} in Fig. (9). The terms proportional to tensor interaction terms contribute destructively to the SM result.

For the $b \to d\tau^+\tau^-$ case, we obtain the following results:

- δA_{CP}^L depends strongly on the tensor type interactions and when C_T is negative (positive) it constructive (destructive) contribution to the SM result. For the other tensor interaction coefficient C_{TE}, the situation is contrary to this behavior (see Figs. (10) and (11)).

- Similar to the μ lepton case, δA_{CP}^L is quite sensitive to the existence of all scalar type interaction coefficients. When the signs of the coefficients C_{LRRL} and C_{LRLR} are negative (positive) the sign of δA_{CP}^L is positive (negative), in the region $\hat{s} \geq 0.6$ (see Figs. (12) and (13)). Note that in the SM case the sign of δA_{CP}^L can be positive or negative. Therefore in the region $\hat{s} \geq 0.6$, determination of the sign of δA_{CP}^L can give unambiguous information about the existence of new physics beyond the SM. For the remaining two scalar interaction coefficients C_{RLRL} (C_{RLLR}), the sign of δA_{CP}^L is negative (positive) (see Figs. (14) and (15)). Again, as in the previous case, determination of the sign and magnitude of δA_{CP}^L can give quite valuable hints for establishing new physics beyond the SM.

Since transversal and normal polarizations are proportional to the lepton mass, for the light lepton case, obviously, departure from the SM results is not substantial for all Wilson coefficients. On the other hand, for the $b \to d\ell^-\ell^+$ transition, δA_{CP}^i ($i = T$ or N) is strongly dependent on C_{LR} (see Fig. (16)), C_{RR} (see Fig. (17)) and scalar type interactions. Note that, the dependence of δA_{CP}^T on C_T and C_{TE} is quite weak.

Finally, we would like to discuss the following question. As has already been mentioned, A_{CP}, as well as δA_{CP}, are very sensitive to the existence of new physics beyond the SM. The intriguing question is that, can we find a region of C_X, in which A_{CP} agrees with the SM result while δA_{CP} does not. A possible existence of such a region will allow us to establish new physics by only measuring δA_{CP}. In order to verify whether such a region of C_X does exist or not, we present the correlations between partially integrated A_{CP} and δA_{CP} in Figs. (18)–(20). The integration region for the $b \rightarrow d\mu^+\mu^-$ transition is chosen to be $1 \ GeV^2/m_b^2 \leq \hat{s} \leq 8 \ GeV^2/m_b^2$, and for the $b \rightarrow d\tau^+\tau^-$ transition it is chosen as $18 \ GeV^2/m_b^2 \leq \hat{s} \leq 1$. These choices of the regions are dictated by the requirement that A_{CP} and δA_{CP} be free of resonance contributions.

In Figs. (18)–(19) we present the correlations δA_{CP}^i and A_{CP}^i asymmetries, when one of the leptons is longitudinally polarized, for the μ and τ lepton cases, respectively. In Fig. (20) we present the flows in the (A_{CP}^T and δA_{CP}^T) plane, when the final lepton is transversally polarized. From these figures we observe that, indeed, there exists a region of new Wilson coefficients in which A_{CP} agrees with the SM prediction, while δA_{CP} does not (in Figs (18)–(20), intersection point of all curves correspond to the SM case).

The numerical values of δA_{CP}^N and A_{CP}^N are very small and for this reason we do not present this correlation. As a final remark we would like to comment that, similar calculation can be carried out for the $B \rightarrow \rho\ell^+\ell^-$ decay in search of new physics, since its detection in the experiments is easier compared to that of the inclusive $b \rightarrow d\ell^+\ell^-$ decay.

In conclusion, we study the CP violating asymmetries, when one the final leptons is polarized, using the most general form of effective Hamiltonian. It is seen that, δA_{CP} and A_{CP} are very sensitive to various new Wilson coefficients. Moreover, we discuss the possibility whether there exist regions of new Wilson coefficients or not, for which A_{CP} coincides with the SM prediction, while δA_{CP} does not. In other words, if there exists such regions of C_X, this means new physics effects can only be established in δA_{CP} measurements. Our results confirm that, such regions of C_X do indeed exist.

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Figure captions

Fig. (1) The dependence of A_{CP} on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}.

Fig. (2) The same as in Fig. (1), but at fixed values of C_{LR}.

Fig. (3) The same as in Fig. (1), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_{LR}.

Fig. (4) The same as in Fig. (3), but at fixed values of C_{LRRL}.

Fig. (5) The same as in Fig. (3), but at fixed values of C_{LRLR}.

Fig. (6) The same as in Fig. (3), but at fixed values of C_T.

Fig. (7) The same as in Fig. (3), but at fixed values of C_{TE}.

Fig. (8) The dependence of δA_{CP}^L on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}, when one of the final leptons is longitudinally polarized.

Fig. (9) The same as in Fig. (8), but at fixed values of C_{LRRL}.

Fig. (10) The same as in Fig. (8), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_T.

Fig. (11) The same as in Fig. (10), but at fixed values of C_{TE}.

Fig. (12) The same as in Fig. (10), but at fixed values of C_{LRRL}.

Fig. (13) The same as in Fig. (10), but at fixed values of C_{LRLR}.

Fig. (14) The same as in Fig. (10), but at fixed values of C_{RLRL}.

Fig. (15) The same as in Fig. (10), but at fixed values of C_{RLLR}.

Fig. (16) The same as in Fig. (10), but when one of the final leptons is transversally polarized, at fixed values of C_{LR}.

Fig. (17) The same as in Fig. (16), but at fixed values of C_{RR}.

Fig. (18) Parametric plot of the correlation between the partially integrated A_{CP}^L and δA_{CP}^L as a function of the new Wilson coefficients C_X, for the $b \to d\mu^+\mu^-$ transition, when one of the final leptons is longitudinally polarized.

Fig. (19) The same as in Fig. (18), but for the $b \to d\tau^+\tau^-$ transition.

Fig. (20) The same as in Fig. (19), but when one of the final leptons is transversally polarized.
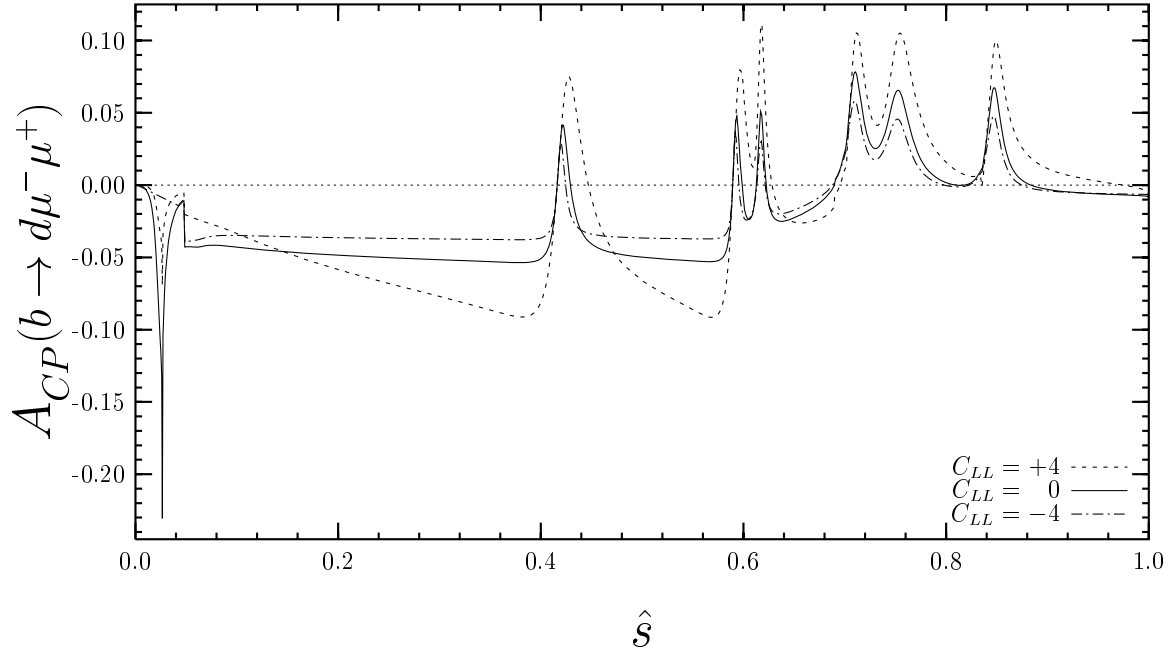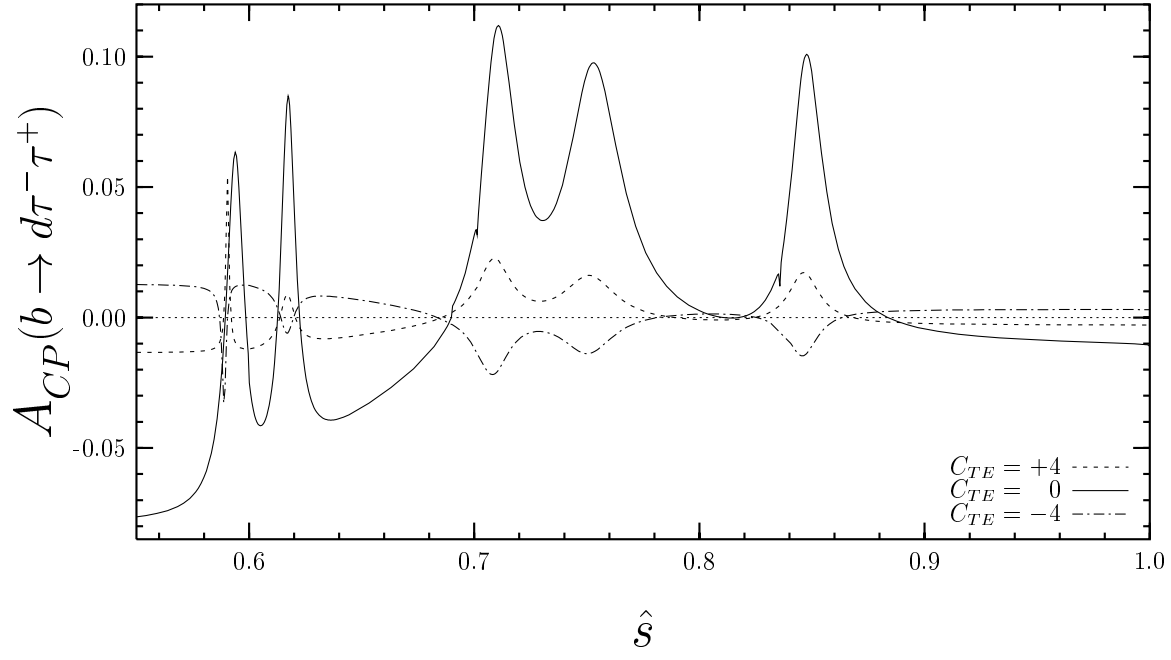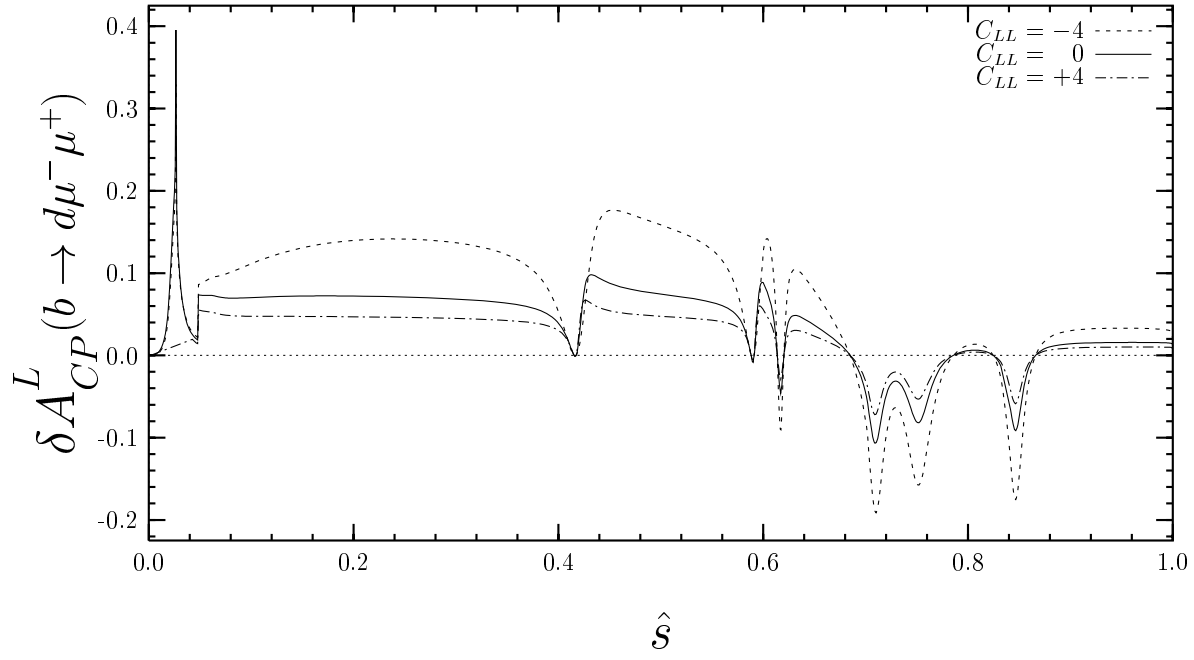
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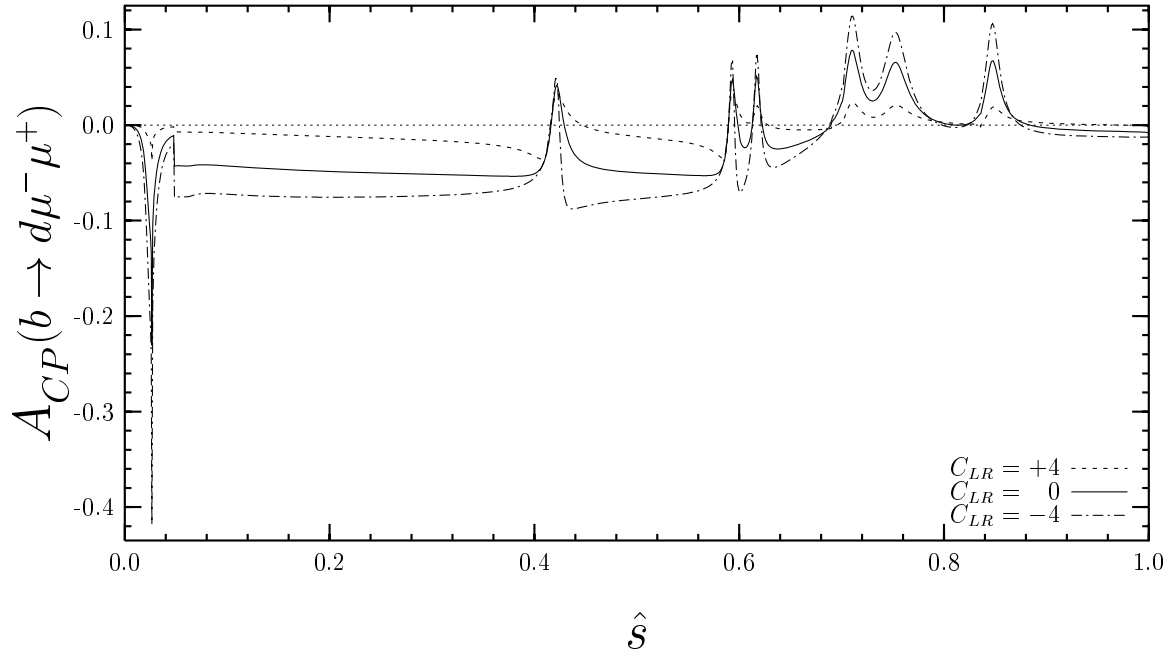
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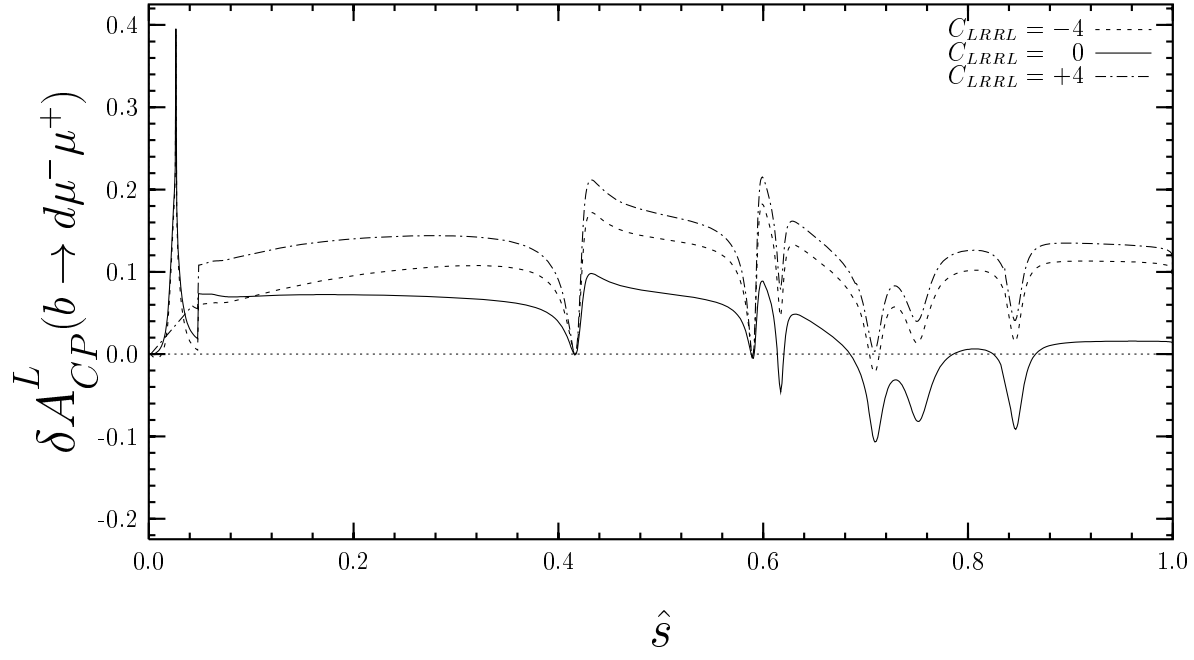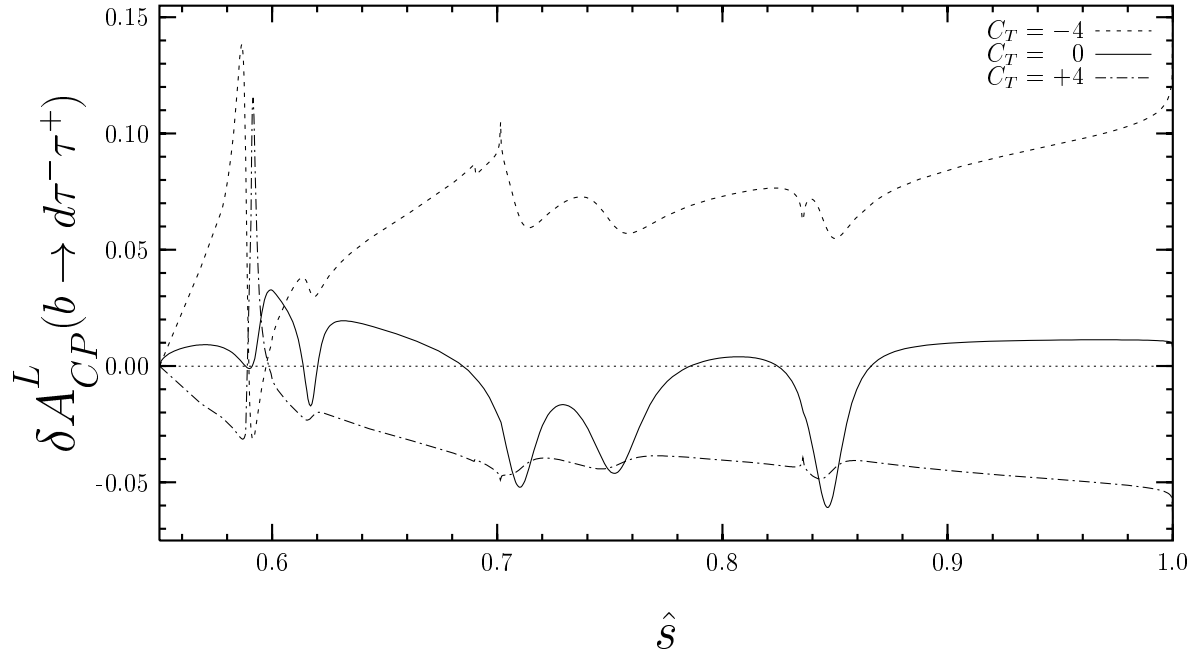
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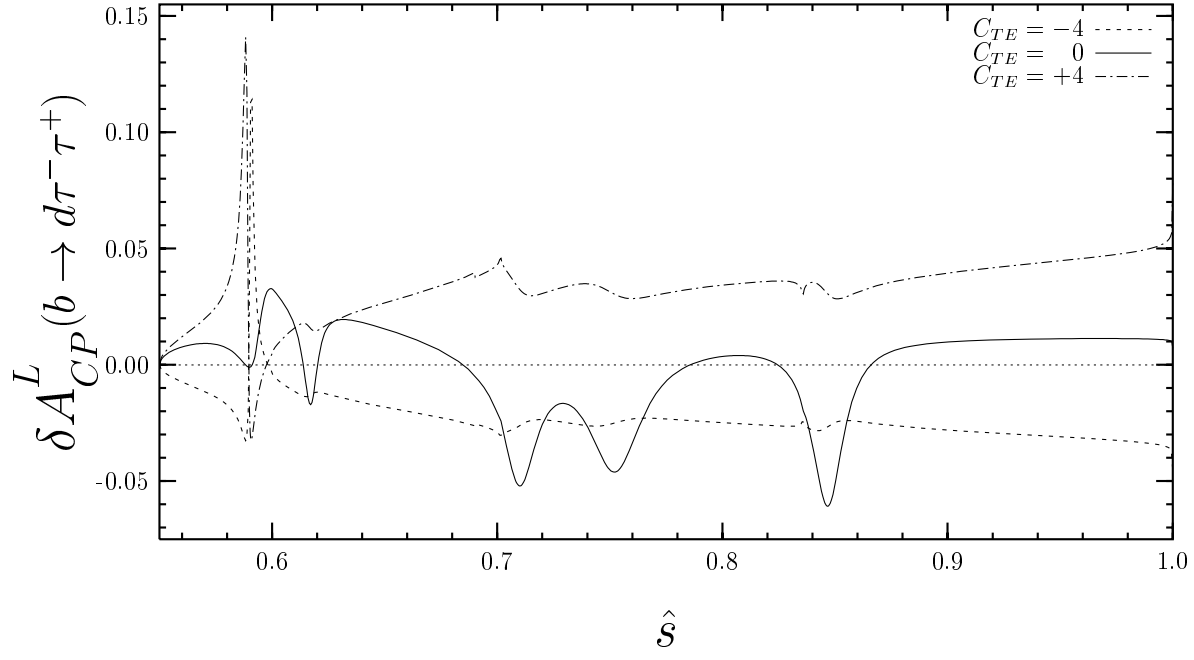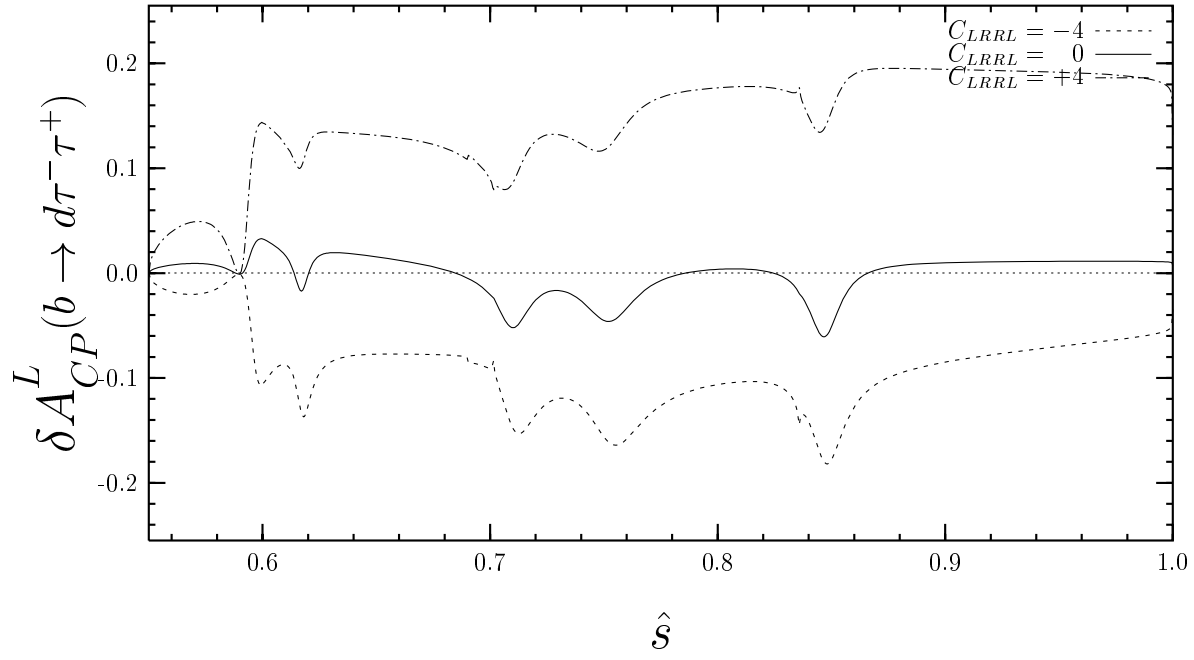Figure 5:

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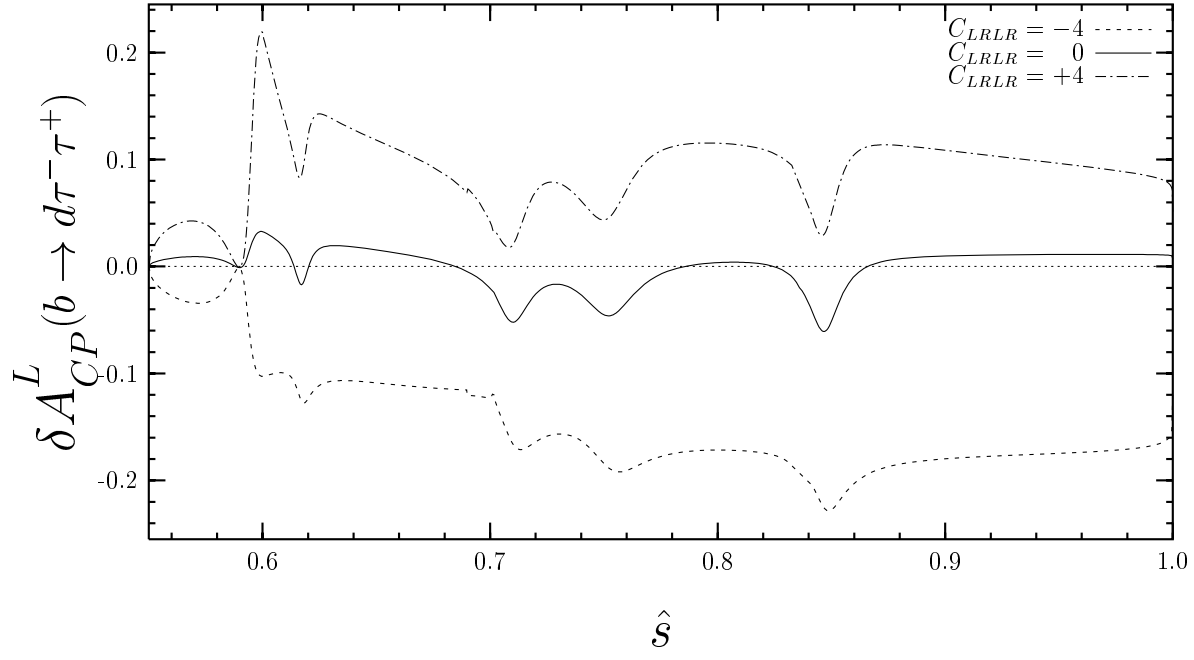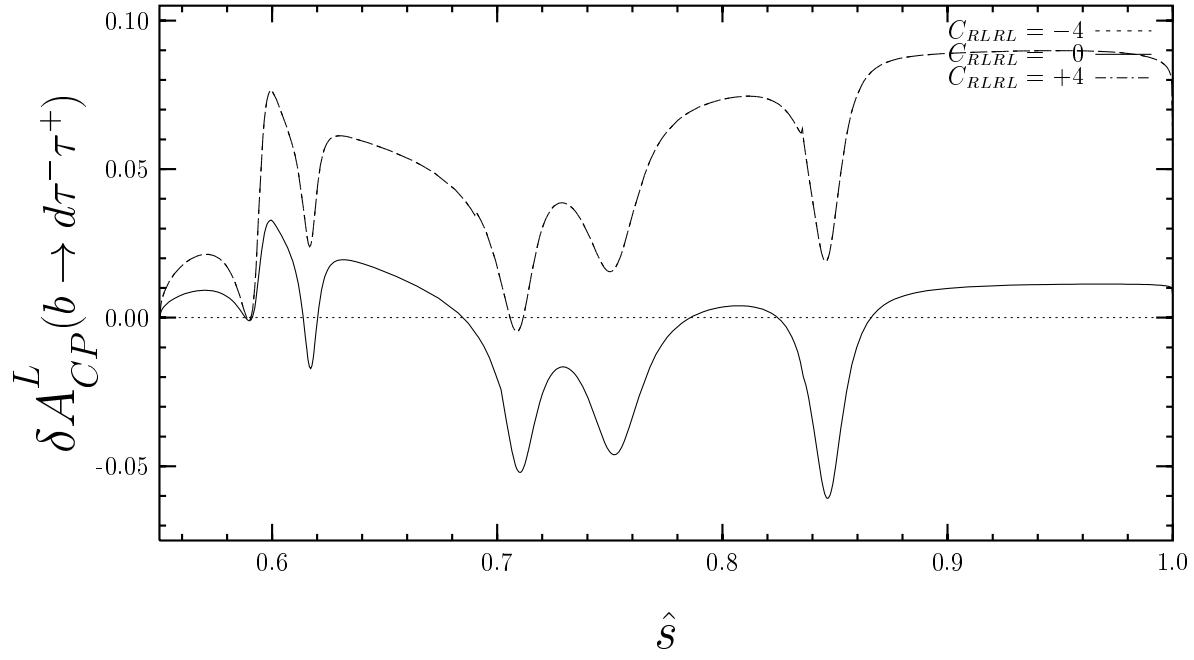Figure 7:

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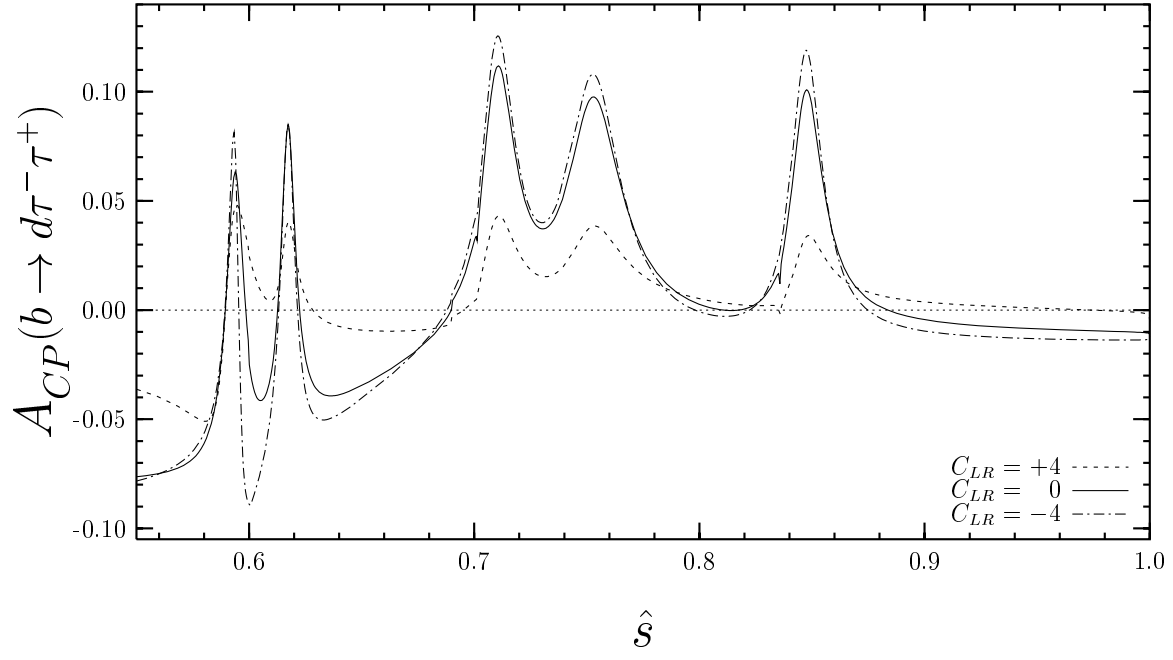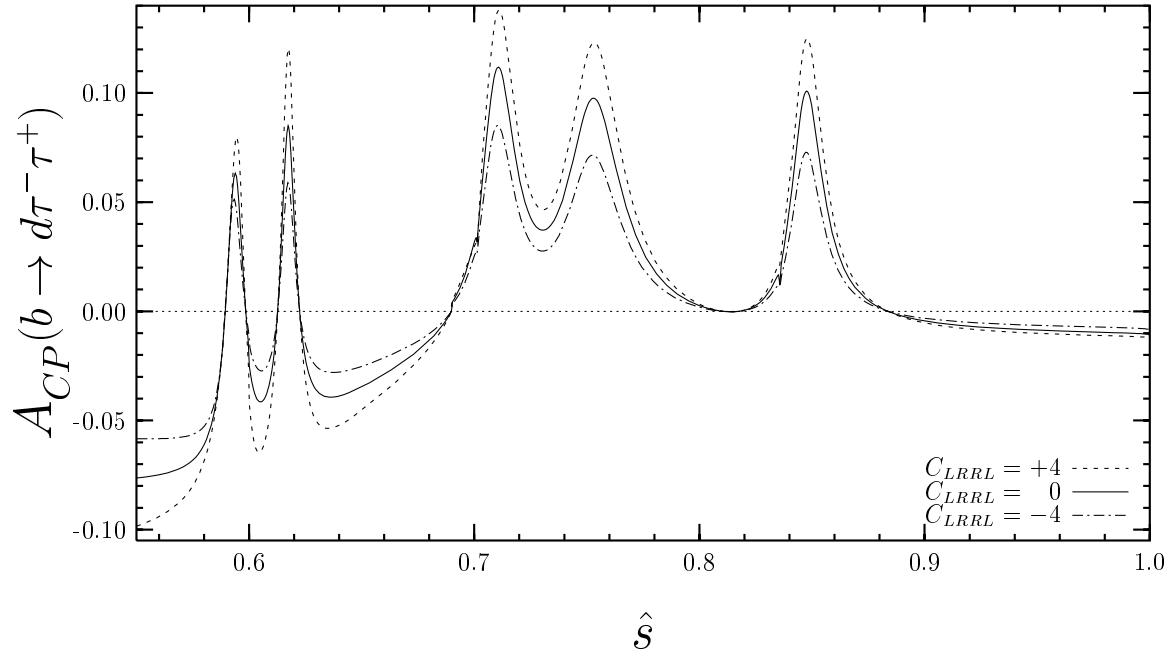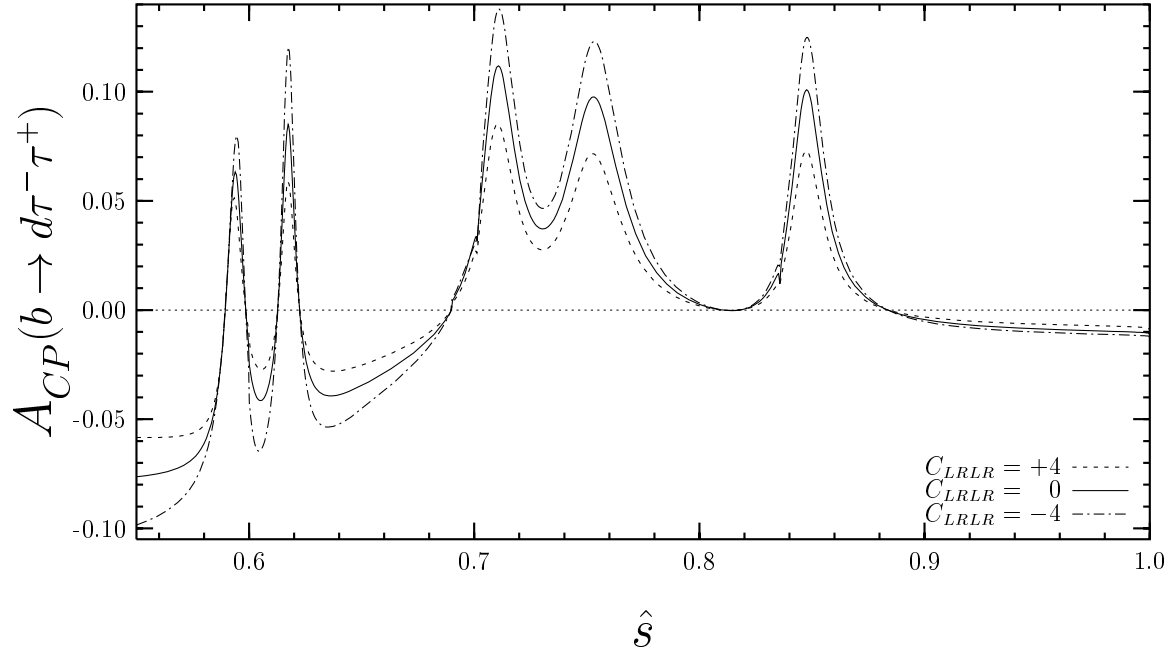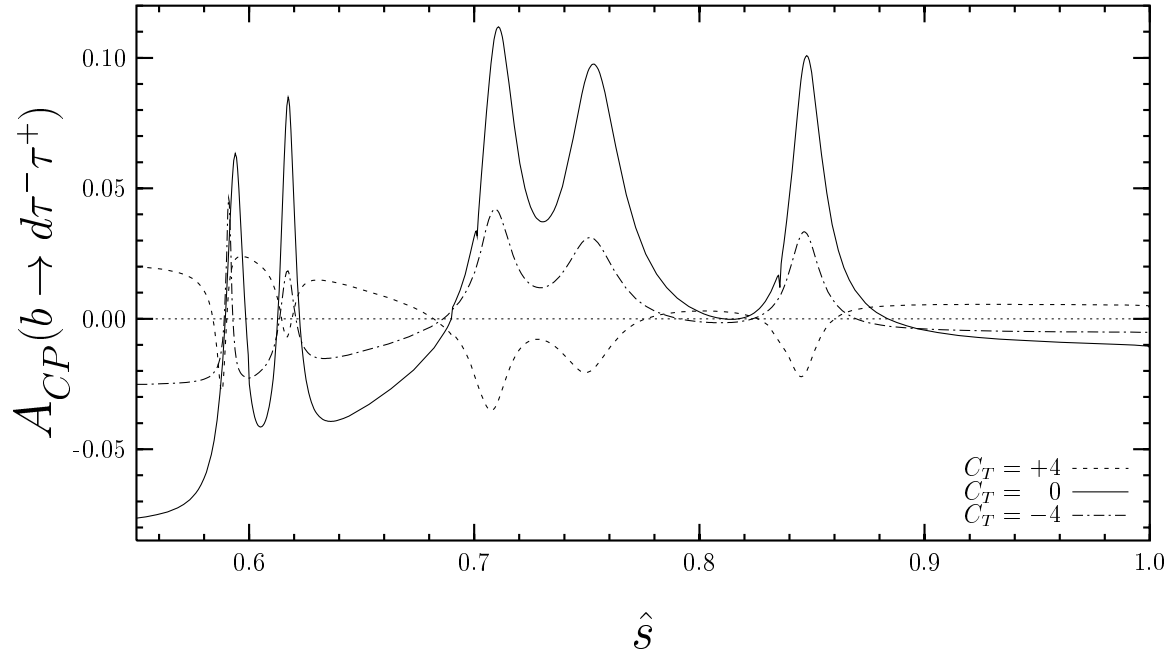
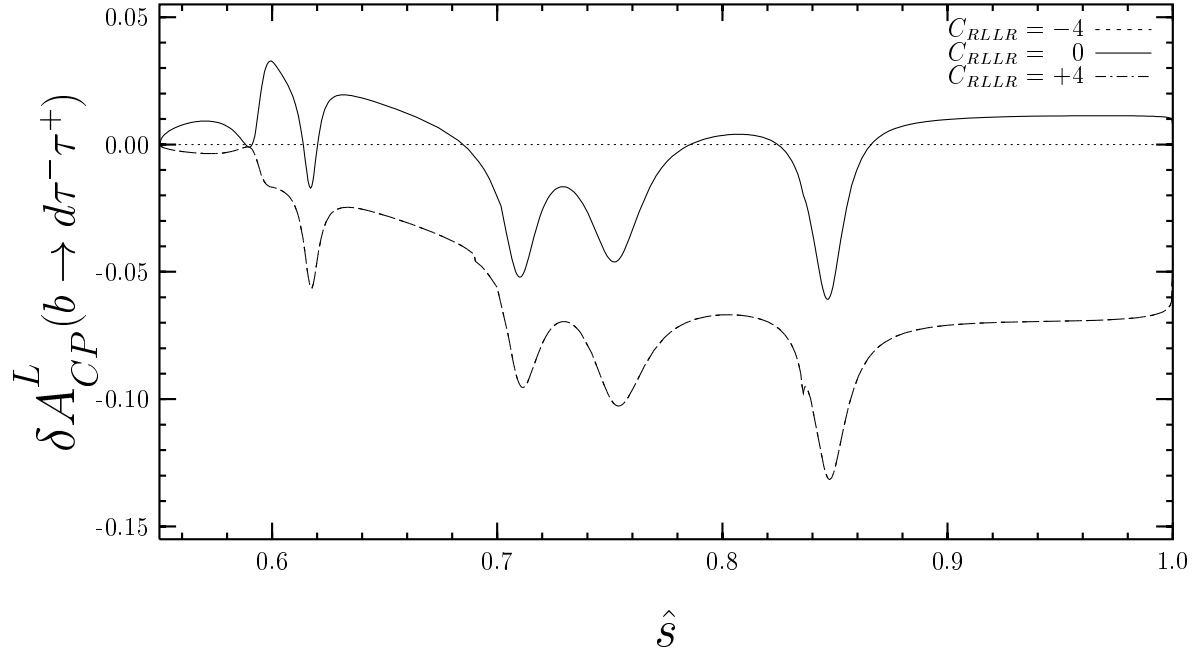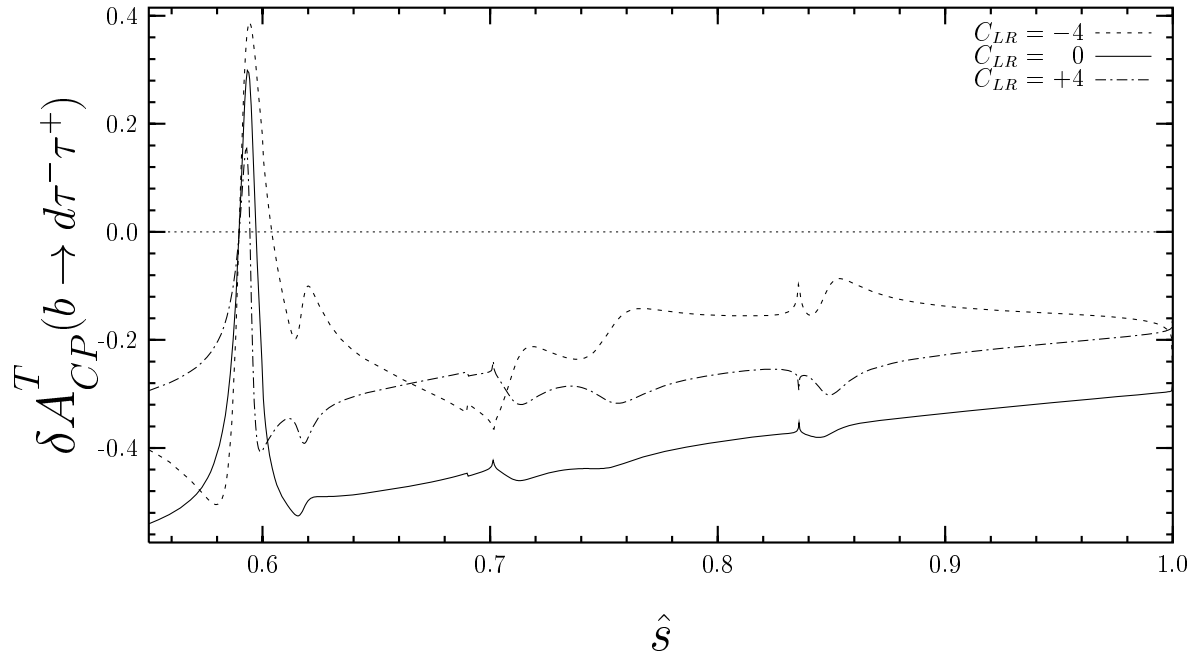Figure 9:

Figure 10:

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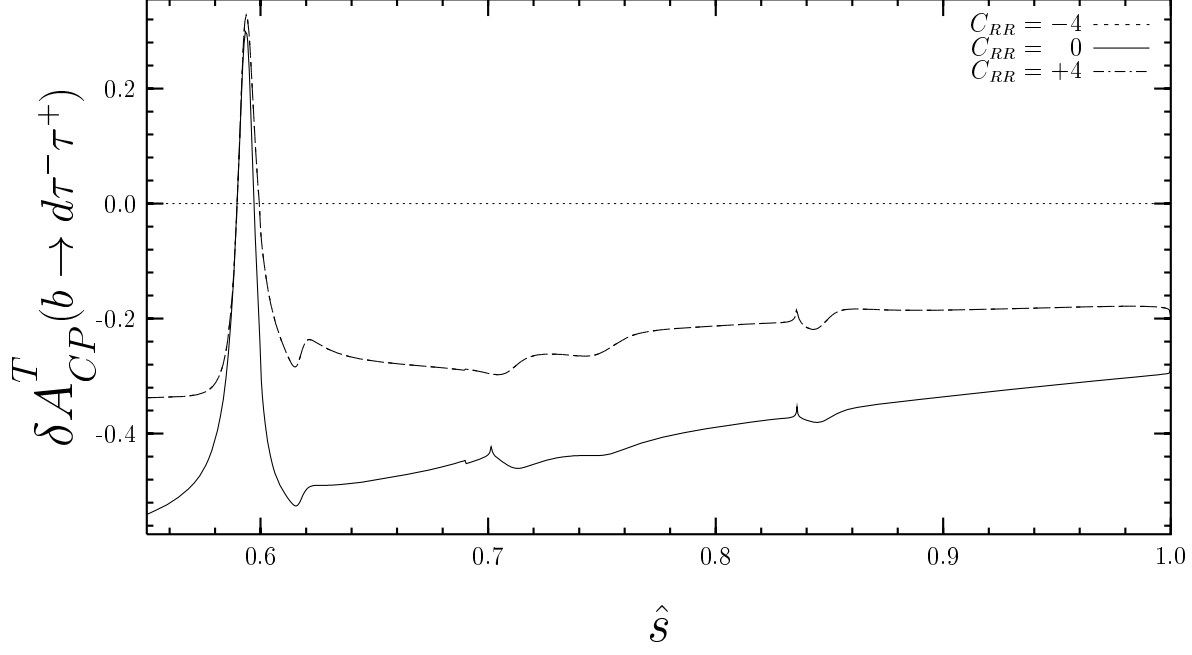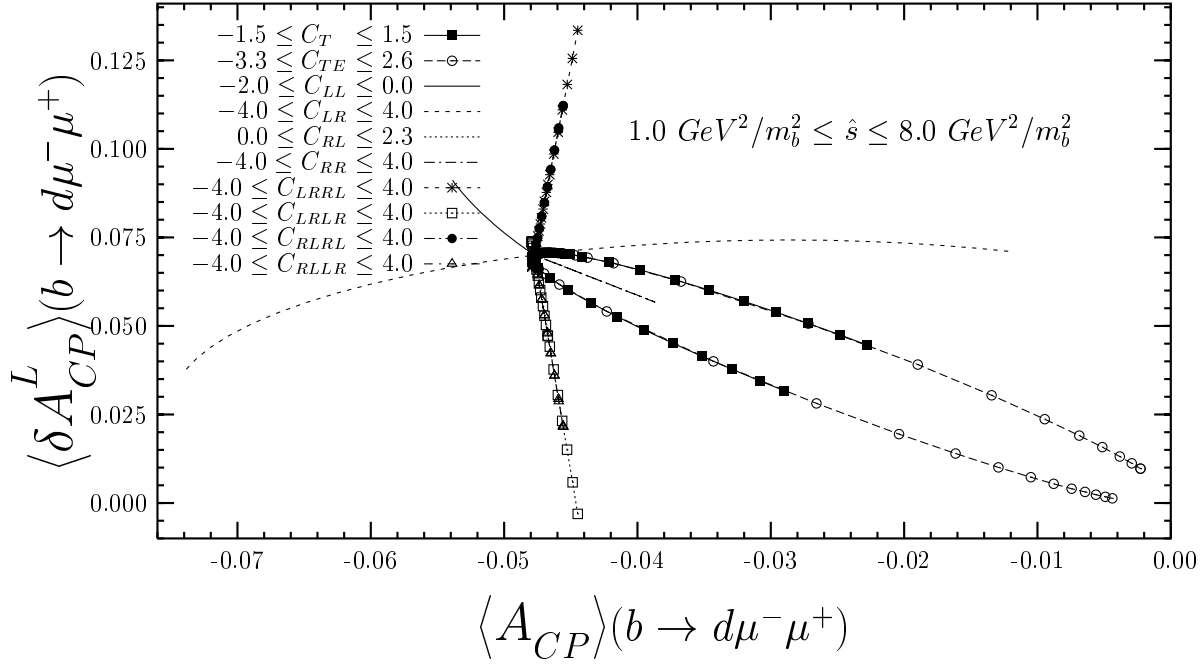Figure 11:

Figure 12:

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Figure 13:

Figure 14:

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Figure 15:

Figure 16:

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Figure 17:

Figure 18:

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Figure 19:

Figure 20:

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